Name: ______________________________

MAC 2312 – Calculus II
First Day Project

General Questions

1. Write the full names of every person in your group. Be sure you can pronounce them all if asked.

2. (a) If you had to guess, how old is your instructor?

   (b) Fill in the blank:
   If my instructor wasn't a mathematician, he'd be ________________________.

   (c) Your instructor likes each of the following bands/artists:

   Gucci Mane, Mumford and Sons, Lana del Rey, Metallica, Elton John.

   Rank the based on how popular *you think* they are to your instructor.

   Most Popular:
   Second:
   Third:
   Fourth:
   Least Popular:

3. You are at an unmarked intersection: In one direction is The City of Lies and in the other is The City of Truth. A citizen from one of those cities (you don't know which) is at the intersection. Given that citizens of The City of Lies always lie and that citizens of the City of Truth always tell the truth, which **one** question could you ask the person to find the way to the City of Lies?

4. (a) List (at least) one cool thing you did this summer.

   (b) List (at least) one cool thing you *learned* this summer that you didn't know before.

5. True or false: Numbers never lie. Justify your answer.

6. What's your favorite number? Favorite math concept? Favorite science concept?

7. What do you want to be *when you grow up*?

Math Questions

8. In your own words, define each of the following math terms:

   (a) Function.

   (b) Polynomial.

   (c) Derivative (of a function).

   (d) Antiderivative (of a function).

   (e) Integral (of a function).

9. e^{π} is larger than π^{e}. Prove it. *Hint: Consider the first derivative of* $f(x) = \dfrac{\ln(x)}{x}$.

10. Let f be the quadratic $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$, $a < 0$.

    (a) Compute $y_0 = f\left(\dfrac{-b}{2a}\right)$. Simplify fully.

    (b) What is the value of y_0 when $b^2 - 4ac = 0$? Interpret this result geometrically.

    (c) Use part (a) to determine the sign of y_0 when $b^2 - 4ac < 0$ and interpret the result geometrically.

11. (a) What is the smallest nonnegative integer?

    (b) What is the smallest positive integer?

    (c) What is the smallest positive real number?

    (d) What is the largest positive real number?

12. Indicate whether each of the following statements is *true* or *false*, assuming that all derivatives/integrals exist and are defined over the indicated domains.

(a) Antiderivatives are unique. ________________

(b) $\int_a^b f(x)\,dx = \int_a^b f(y)\,dy.$ ________________

(c) If F is an antiderivative for f, then $\int_a^b f(x)\,dx = F(b) - F(a).$

____

(d) The tangent line to the graph of a function f at a point x_0 always intersects the graph exactly once.

____

(e) $\int f(x)\,dx = \int f(y)\,dy.$ ________________

(f) If $F(x) = \int_a^x f(t)\,dt$ is any antiderivative of f, then $G(x) = F(x)+C$ is another antiderivative of f for any real number C.

____

(g) $\frac{d}{dy}\int f(y)\,dy = f(y).$ ________________

(h) $\lim_{x\to 0} \frac{\sin x}{x} = 1$ ________________

(i) If f is an odd function and F is an antiderivative for f, then F is an odd function.

____

(j) If f is an odd function, then its derivative f' is an odd function.

____

(k) $\frac{d}{dx}\left(x^x\right) = x^x + x^x \ln(x).$ ________________

(l) For a function g which is increasing on $[a,b]$, the approximation of $\int_a^b g(x)\,dx$ given by a *right* Riemann sum is an *underestimate* of the actual value.

____

(m) Every differentiable function is continuous. ____________________

(n) The second derivative test says that if $f'(x_0 - \Delta x) > 0$ and $f'(x_0 + \Delta x) < 0$, then x_0 is a maximum for f.

____

(o) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. ____________________

(p) If a function h changes concavity at a point x, then $h''(x) = 0$.

____

(q) If a function h satisfies $h''(x) = 0$ for some x, then h changes concavity at x.

____

(r) The tangent line to the graph of a function f at a point x_0 always intersects the graph at least once.

____

(s) $\int \ln(x)\,dx = x\ln(x) + C$. ____________________

(t) If $p(x)$ is a polynomial of degree n, then p can have at most n relative extrema.

____

(u) If $p(x)$ is a polynomial of degree n, then p can have at most n *absolute* extrema on any closed interval $[a,b]$.

____

(v) $\frac{d}{dx}\pi^x = x\pi^{x-1}$. ____________________

(w) $\frac{d}{dx}x^e = ex^{e-1}$. ____________________

(x) $\frac{d}{dx}e^x = e^x$. ____________________

(y) $\frac{d}{dx}e^\pi = \pi e^{\pi-1}$. ____________________